校長雙語公開觀課-前言

CCHS

To strengthen the competitiveness of the young generation in Taiwan and equip them with the ability to obtain better employment opportunities and salary income, Taiwan government actively promotes bilingual policy. The two major goals of the policy are "enhancing English proficiency " and "enhancing national competitiveness“. By 2023, Taiwan will become a bilingual country.

Last year, our school was approved by the Ministry of Education to set up a bilingual experimental class. In the future, bilingual teaching will be the mainstream of our school, and the students will be able to use the two languages naturally in their daily lives.

To encourage the teachers of different subjects to confidently add English to their teaching, today, I, the principal, set an example by giving bilingual mathematics lessons, hoping that everyone can contribute to bilingual education of Ching Cheng High School.

Let us cultivate students' international competitiveness and communication skills together.

今天要講授的內容是高一數學第二冊~直線排列

Today's lecture is the second volume of Mathematics for the first year of senior high school students~linear permutations

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111學年度校長公開觀課

科目：高中數學(雙語教學)
範圍：高一(二)
內容：直線(相異物＆具相同物)排列

Permutations

Multiplication Rule

If one event can occur in m ways, a second event in n ways and a third event in r, then the three events can occur in $m \times n \times r$ ways.

Permutations

- An arrangement of a set of objects is called a permutation.
- The word permutation comes from the Latin words per + mutare that together mean “by change” or “through change.”
- **TYPE1**：linear arrangement**(直線排列)**

  In how many ways can 6 people if only 3 of them are chosen arranged in a row?

- **TYPE2**：**Repetition allowed(重複排列-**repeated permutation)

  If one event with n outcomes occurs r times with repetition allowed, then the number of ordered arrangements is ?

Mathematical calculation symbols of permutations

Permutations of Different Objects

Example

Chico has 7 songs on his iPod.He has time to listen
(1) to 3 songs on his way to school.
(2) to all the 7 songs on his way to school
How many arrangements different songs are possible?

Conclusion

$$n! = n(n-1)(n-2)\cdot\ ...\ \cdot 3\cdot 2\cdot 1$$
$$0! = 1$$

$n!$ represents the number of permutations of n different objects.

Permutations of Different Objects

The number of permutations of n distinct objects taken r at a time is:

$${}_nP_r = \frac{n!}{(n-r)!},\ n \geq r$$

Example 1

Find the value of *n* such that

(i) $P_5^n = 42P_3^n$, $n>4$ (ii) $\boldsymbol{P_n^4 : P_{n-1}^5 = 6:5.}$

Example 2

用六個數字0、1、2、3、4、5組成沒有重複數字的三位數

(1)能組成多少個三位數?

(2)這些三位數中不是5的倍數者有多少個?

Example 3

During the graduation trip, five students, A, B, C, D, and E, lined up and entered the haunted house of playground. In how many of these arrangements:

某次畢業旅行有甲、乙、丙、丁、戊等五位同學排成一行進入遊樂場鬼屋，試問下列各小題有幾種排列方式：

(1) How many arrangements are there for E not going to the front?

(2) How many arrangements are there in which E does not go to the front and C does not go to the back?

(3) A is the girlfriend of B, how many ways are there to arrange that they must walk together (adjacent to each other)?

(4) How many ways are there in which A and B hate each other and must not walk together (not adjacent to each other)?

Permutations - linear arrangement(直線排列)

Permutations of Identical Objects(具相同物)

the permutation problems involved objects that were different. Sometimes some of the objects are identical.

Consider the 2types of 3 letters DEE and DE_1E_2, the number of arrangements would be:

However, each word contains 2 Es. In each word, when the Es are interchanged, the result is the same word.

There are ways of arranging E_1 and E_2

So, the number of arrangements becomes: $\frac{4!}{2!} = 12$

The number of permutations of n objects with r identical objects is: $\frac{n!}{r!}$

Permutations - linear arrangement(直線排列)

And so on, Consider a collection of 3 identical soccer balls, 2 identical baseballs, and 1 basketball. If all the sports balls were different, they could be arranged in a row

the number of ways to arrange the sports balls is:

$$\frac{6!}{3!2!1!}=60$$

Permutations of Objects of Multiple Kinds

In a set of n objects with n_1 objects of one kind, n_2 objects of another kind, n_3 objects of another kind, and so on, for k kinds of objects, the number of permutations is:

$$\frac{n!}{n_1!n_2!n_3!\ldots n_k!}, \text{ where } n_1 + n_2 + n_3 + \ldots + n_k = n$$

Permutations - linear arrangement(直線排列)

Example 1

*The program list of **5 programs originally scheduled** for a certain class's birthday party has been scheduled. If **two new programs are temporarily added**, and the corresponding order of the original **5 programs remains unchanged**, the program list after adding the two new programs can be how many kinds?*

Example 2

A、B、C、D、E、F six letters in a row.

(1) If both A and B need to be on the same side of C

(2) If A is to the right of B and C is to the right of D

Example 3 - 捷徑問題

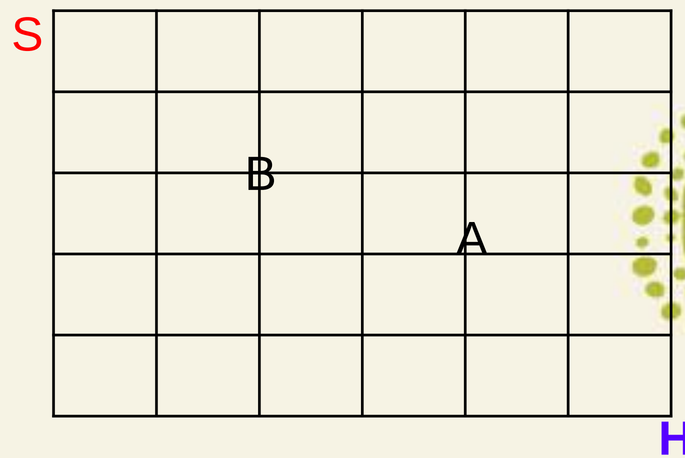
Abby walks 9 blocks from her home, H, to school, S. She always walks 5 blocks west and 4 blocks north.

(1) How many ways can Abby walk to school?

(2) How many shortest paths are there through A?

(3) How many shortest paths are there that pass through A and pass through B?

(4) How many shortest paths are there that do not go through A and do not go through B?



Example 4

A frog stands at the origin, Jump left or right one unit long with each step, It is known that the frog returns to the original point after jumping six steps. How many ways can the frog jump?

Exercise

1.甲、乙、丙、丁、戊、己、庚共七人排成一列，試求下列各小題的排列數:

(1) 任意排　(2) 甲乙必須相鄰　(3) 甲排在乙的前面

(4) 甲乙必須分開　(5) 甲不排首，乙不排第二位，丙不排第三位

(6) 甲乙丙都在丁的前面　(7) 甲丙在戊的前面，庚在乙的後面

2.Known fire alarms have two types of long and short beeps. It takes 3 seconds for a long beep, 1 second for a short beep, and a 2-second pause between two beeps. If the total sounding time is 30 seconds, how many kinds of alarm signals can be sent in total?

NOTE 1：簡單邏輯logic與集合set

- 敘述&邏輯-- 否定敘述
- 集合運算operation與計數counting

- $(A\cap B)'=A'\cup B'$ ； $(A\cup B)'=A'\cap B'$
- $n(A\cup B)=n(A)+n(B)-n(A\cap B)$
- $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$

NOTE 2：直線排列

- **完全相異物的直線排列linear arrangement**

  The number of permutations of n different objects taken r at a time, where0<$r\delta n$ and the objects do not repeat is $n(n–1)(n–2)\ldots(n–r+1)$, which is denoted by $p_r^n=\frac{n!}{r!}$.

- **具相同物的直線排列**

  The number of permutations of n objects, where r_1 objects are of one kind, r_2 are of second kind, ..., r_k are of kth kind and the rest, if any, are of different kind is $\frac{n!}{r_1!r_2!\ldots rk!}$, where $n=r_1+r_2+\cdots+rk$

- **常見的限制或條件**

  定位&不排;相鄰&分開;前後或左右;捷徑問題