

Arithmetic Sequences Problems

P18

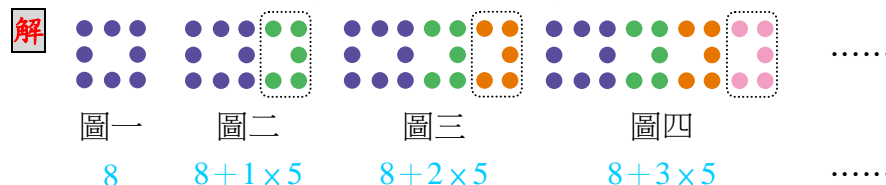
例 6 等差數列的一般項

搭配習作 P4 基礎題 3 自評 P31 第 3 題

下面各圖是由圓點「●」所組成的規律圖形：



若圖 n 中，圓點的總數為 a_n ，以 n 的式子表示 a_n 。



由圖形的規律發現，依序會增加 5 個圓點，

可知 a_1, a_2, \dots, a_n 為一等差數列，其首項為 8，公差為 5，

所以 $a_n = 8 + (n-1) \times 5 = 5n + 3$ 。

翻譯示例：

Example 6

The general term of an arithmetic sequence.

The following figures are regular figures composed of dots "●":

If the number of dots in Figure n is a_n , express a_n , (a sub n) as a function (or a formula) of n .

Solution

From the rule of the figure, we can find that five dots will be added in the sequence to each term.

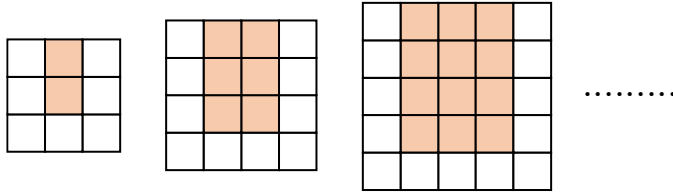
It can be seen that a_1, a_2, \dots, a_n is an arithmetic sequence,

the first term is 8, and the common difference is 5,

so $a_n = 8 + (n-1) \times 5 = 5n + 3$.

隨堂練習

如下圖，將白色方塊與橘色方塊按照規律拼成若干個正方形圖案。其中的橘色方塊構成一個長方形，且長方形各邊的方塊數每次都會增加一個。設 a_n 為圖 n 中 U 字型白色方塊的總數，以 n 的式子表示 a_n 。



圖一

圖二

圖三

由圖形的規律發現，依序會增加 3 個白色方塊，其首項為 7，公差為 3，
所以 $a_n = 7 + (n-1) \times 3 = 3n + 4$ 。

翻譯示例：

Practice

The following figure shows white and orange squares combined into several square figures according to specific rules.

The orange squares form a rectangle, and the number of squares on each side of the rectangle will increase by one each time.

Let a_n be the total number of white squares with a U font in Figure n , and let a_n , (a sub n) be a function of n .

From the rule of the figure, we can find that three white squares will be added in sequence,

the first term is 7, and the common difference is 3,

$$a_n = 7 + (n-1) \times 3 = 3n + 4.$$

P19**例 7 存款問題**

自評 P32 第 5 題

3 月 1 日傑克已有存款 350 元，他自 3 月 2 日起，每日皆再儲蓄 55 元，則幾月幾日傑克才有足夠的錢購買 2000 元的玩具？

解

設自 3 月 1 日算起第 n 天，傑克才有足夠的錢購買 2000 元的玩具。

傑克每日存款總額成等差數列 350, 405, 460, ……，此等差數列的首項 $a_1 = 350$ ，公差 $d = 55$ 。

$$\text{則 } a_n = a_1 + (n-1)d \geq 2000$$

$$350 + (n-1) \times 55 \geq 2000$$

$$55n \geq 1705$$

$$n \geq 31$$

所以在 3 月 31 日時，傑克的存款總額剛好可以購買 2000 元的玩具。



翻譯示例：

Example 7**Savings Account Question**

Jack deposits NT \$350 into an account on March 1.

Starting on March 2nd, he deposits NT \$55 into the same account each day. On what date will Jack have enough money to buy a toy which is worth NT\$2000?

Solution

Suppose Jack has enough money to buy a toy which is worth NT\$2000. Then n days after March 1, the total amount of Jack's daily deposit can form an arithmetic sequence :

350, 405, 460, ...,

The first term $a_1 = 350$, and the common difference $d = 55$.

$$\text{Then } a_n = a_1 + (n-1)d \geq 2000$$

$$350 + (n-1) \times 55 \geq 2000$$

$$55n \geq 1705$$

$$n \geq 31$$

So on March 31, Jack's total deposit is just enough to buy a toy which is worth NT\$2000.

隨堂練習

承例 7，若傑克想買 3000 元的玩具，要存到幾月幾日，他才有足夠的錢購買？
 設自 3 月 1 日算起第 n 天，傑克才有足夠的錢購買 3000 元的玩具。

$$350 + (n-1) \times 55 \geq 3000$$

$$55n \geq 2705$$

$$n \geq \frac{2705}{55}$$

$$n \geq 49\frac{2}{11}$$

即 $n=50$

$$50 - 31 = 19$$

所以 4 月 19 日時，傑克才有足夠的錢購買 3000 元的玩具。

解答：67。(相鄰兩項的差為等差數列)

翻譯示例：

Practice

Follow example 7. If Jack wants to buy a toy that is worth NT\$3000, on what date will he have enough money to buy it?

Solution

Suppose n days after March 1, Jack has enough money to buy a toy which is worth NT\$3000.

$$350 + (n-1) \times 55 \geq 3000$$

$$55n \geq 2705$$

$$n \geq \frac{2705}{55}$$

$$n \geq 49\frac{2}{11}$$

$$n=50$$

$$50 - 31 = 19$$

So Jack has enough money to buy a toy that is worth NT\$3000 on April 19.

Answer: 67. (the difference between two adjacent terms is an arithmetic sequence)

P20**►等差中項**

如果 a, b, c 三數成等差數列，則 b 稱為 a 與 c 的**等差中項**。

例如： $-8, 12, 32$ 成等差數列，則 12 為 -8 與 32 的等差中項。

因此，若 a, b, c 三數成等差數列，則 $b - a = c - b$ ← 後項減前項等於公差

$$2b = a + c$$

$$b = \frac{a+c}{2}$$

【等差中項公式】

b 為 a 與 c 的等差中項，則 $2b = a + c$ ，即 $b = \frac{a+c}{2}$ 。

翻譯示例：

►Median of an arithmetic sequence

If the three numbers a, b , and c form an arithmetic sequence,
then b is the median of an arithmetic sequence of a and c .

For example, if $-8, 12$, and 32 form an arithmetic sequence,
then 12 is the median of an arithmetic sequence of -8 and 32 .

Therefore, if the three numbers a, b , and c form an arithmetic sequence,
then $b - a = c - b$ ← the subsequent term minus the previous term is equal to the common difference

$$2b = a + c$$

$$b = \frac{a+c}{2}$$

【Median of an arithmetic sequence formula】

If b is the median of an arithmetic sequence of a and c ,

Then $2b = a + c$, which means $b = \frac{a+c}{2}$.

例 8 等差中項的應用

搭配習作 P5 基礎題 4 自評 P31 第 4 題

已知三數成等差數列，且其等差中項為 7，求此三數的和。

解

設此三數為 $a, 7, b$ ，因為 $a, 7, b$ 三數成等差數列，所以 $a + b = 2 \times 7 = 14$ 。

因此，三數的和 $a + 7 + b = a + b + 7$

$$= 14 + 7$$

$$= 21$$

翻譯示例：

Example 8

Application of the median of an arithmetic sequence

Given that three numbers form an arithmetic sequence,
and the median of the arithmetic sequence is 7.

Find the sum of these three numbers.

Solution

Suppose these three numbers are $a, 7$, and b .

Since the three numbers $a, 7$, and b are an arithmetic sequence, $a + b = 2 \times 7 = 14$.

Therefore, the sum of three numbers $a + 7 + b = a + b + 7$

$$= 14 + 7$$

$$= 21$$

隨堂練習

已知三數成等差數列，且此三數總和為 -45 ，求此三數的等差中項為多少？

設三數為 a, b, c ，則 $a + b + c = -45$ 。

又 $a + c = 2b$ 代入得 $a + b + c = -45$

$$2b + b = -45$$

$$3b = -45$$

$$b = -15$$

翻譯示例：

Practice

Given that three numbers form an arithmetic sequence,
and the sum of these three numbers is -45 .

Find the median of the arithmetic sequence.

Solution

Suppose these three numbers are a, b , and c .

Then $a + b + c = -45$.

Let $a + c = 2b$ plug in $a + b + c = -45$

$$2b + b = -45$$

$$3b = -45$$

$$b = -15$$

參考資料來源

1. 110 國中數學 2 下翰林版課本
2. IB Maths SL Book Oxford
Chapter 6 Patterns, sequences and series
3. Holt McDougal Larson Algebra 2
Chapter 7 Sequences and Series

☆老師們可以自己從中選擇以做出適合自己學生程度的學習單或是在課堂中適時補充這些英文。

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